2023 AMC 12A Problems

Problem 1

Cities A and B are 45 miles apart. Alice and Barbara start biking from A and B at speeds of 18 mph and 12 mph, respectively. How far away from city A will they be when they meet?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 27$

Problem 2

The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

(A) $1\frac{4}{5}$ (B) 2 (C) $2\frac{2}{5}$ (D) 3 (E) $3\frac{3}{5}$

Problem 3

How many positive perfect squares less than 2023 are divisible by 5?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Problem 4

How many digits are in the base-ten representation of $\mathbf{8^5 \cdot 5^{10} \cdot 15^5}$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

Problem 5

Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

(A) $\frac{2}{9}$ **(B)** $\frac{49}{216}$ **(C)** $\frac{25}{108}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{13}{54}$

Problem 6

Points A and B lie on the graph of $y = \log_2 x$. The midpoint of $\overline{AB}$ is $(6, 2)$. What is the positive difference between the x-coordinates of A and B?

(A) $2\sqrt{11}$ **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** $4\sqrt{5}$ **(E)** 9

Problem 7

A digital display shows the current date as an 8-digit integer, consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For how many dates in 2023 will each digit appear an even number of times in the digital display for that date?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

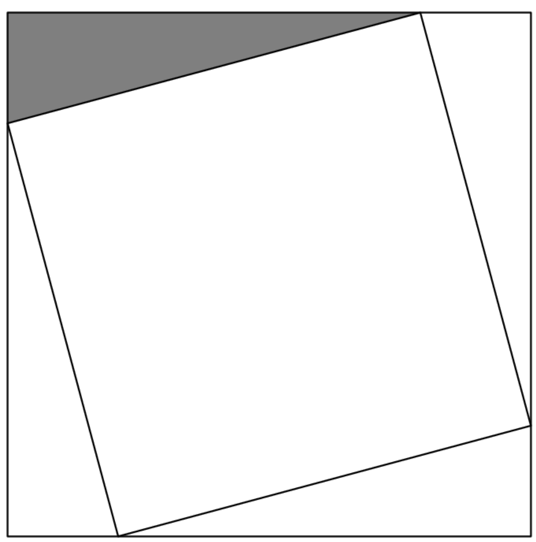
Problem 8

If Mareen scores an 11 on her next test, her mean score will go up by 1. If she gets three 11's in a row, her mean score will increase by 2. What is her current mean test score?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 9

A square with area 3 has a square with area 2 inscribed in it. This creates 4 smaller congruent right triangles. What is the ratio of the smaller leg to the larger leg in the shaded right triangle?



$\textbf{(A)}\ \frac{1}{5} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ 2-\sqrt{3} \qquad \textbf{(D)}\ \sqrt{3}-\sqrt{2} \qquad \textbf{(E)}\ \sqrt{2}-1$

Problem 10

Positive real numbers x and y satisfy

$$y^3 = x^2 \text{ and } (y-x)^2 = 4y^2.$$

What is $x+y$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 40

Problem 11

What is the degree measure of the acute angle formed by lines with slopes 2 and $\frac{1}{3}$?

(A) 30 **(B)** 37.5 **(C)** 45 **(D)** 52.5 **(E)** 60

Problem 12

What is the value of

$$2^3 - 1^2 + 4^3 - 3^3 + 6^3 - 5^3 + \cdots 18^3 - 17^3?$$

(A) 2023 **(B)** 2679 **(C)** 2941 **(D)** 3159 **(E)** 3235

Problem 13

In a tennis tournament, each person plays every other person once. In this tournament, there are twice as many right-handed players than left-handed players, but left-handed players won 40% more games than right-handed players. How many total games were played?

(A) 15 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 66

Problem 14

How many complex numbers z satisfy $z^5 = \overline{z}$, where $\overline{z}$ is the conjugate of the complex number z?

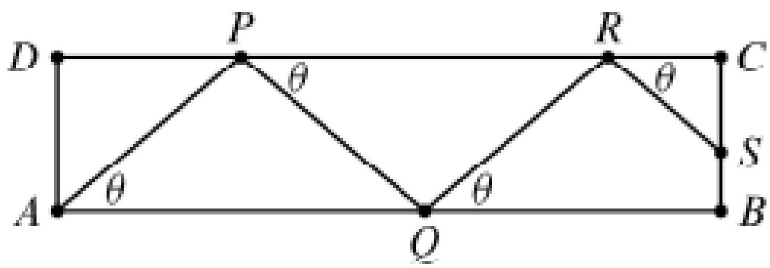
(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

Problem 15

Usain is walking for exercise by zigzagging across a 100-meter by 30-meter rectangular field, beginning at point A and ending on $\overline{BC}$. He wants to increase the distance walked by zigzagging as shown in the figure below ($APQRS$). What angle

$$\theta = \angle PAB = \angle QPC = \angle RQB = \cdots$$

will produce a length that is 120 meters? (The figure is not drawn to scale. Do not assume that the zigzag path has exactly four segments as shown; there could be more or fewer.)



(A) $\arccos\frac{5}{6}$ **(B)** $\arccos\frac{4}{5}$ **(C)** $\arccos\frac{3}{10}$ **(D)** $\arcsin\frac{4}{5}$ **(E)** $\arcsin\frac{5}{6}$

Problem 16

Consider the set of complex numbers z satisfying $|1+z+z^2|=4$. The largest possible value for the imaginary part of z can be written as $\frac{\sqrt{m}}{n}$, where m and n are positive integers and m is not divisible by the square of any prime. What is $m+n$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

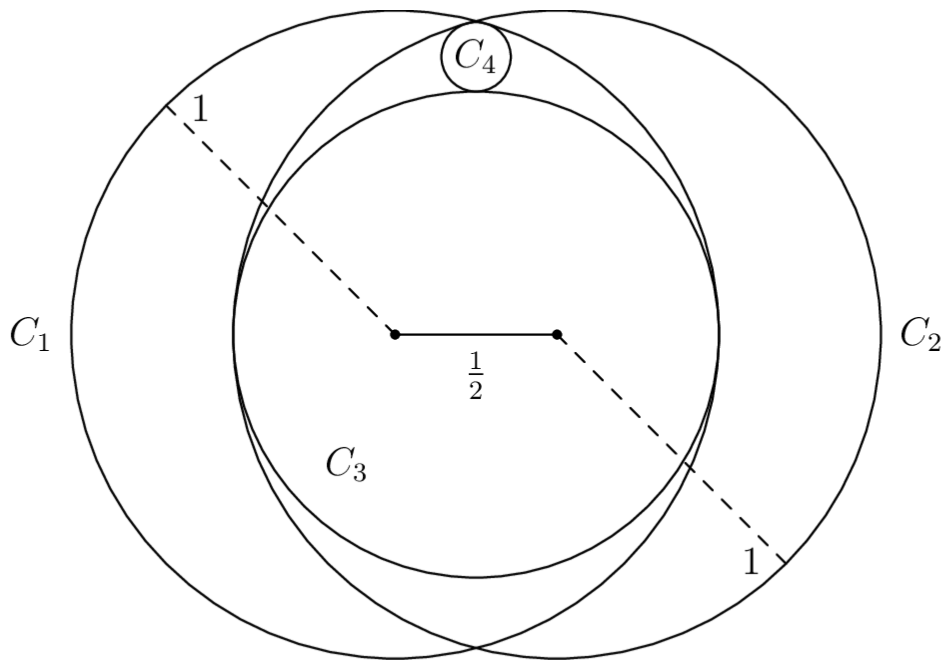
Problem 17

Flora the frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, Flora leaps a positive integer distance m with probability $\frac{1}{2^m}$. What is the probability that Flora will eventually land at 10?

(A) $\frac{5}{512}$ **(B)** $\frac{45}{1024}$ **(C)** $\frac{127}{1024}$ **(D)** $\frac{511}{1024}$ **(E)** $\frac{1}{2}$

Problem 18

Circle C_1 and C_2 have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and is externally tangent to C_3. What is the radius of C_4?



(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

Problem 19

What is the product of all the solutions to the equation

$$\log_{7x} 2023 \cdot \log_{289} 2023 = \log_{2023} 2023?$$

(A) $(\log_{2023} 7 \cdot \log_{2023} 289)^2$ **(B)** $\log_{2023} 7 \cdot \log_{2023} 289$ **(C)** 1

(D) $\log_7 2023 \cdot \log_{289} 2023$ **(E)** $(\log_7 2023 \cdot \log_{289} 2023)^2$

Problem 20

Rows 1, 2, 3, 4, and 5 of a triangular array of integers are shown below:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 3 & & 1 & & \\ & 1 & & 5 & & 5 & & 1 & \\ 1 & & 7 & & 11 & & 7 & & 1 \end{array}$$

Each row after the first row is formed by placing a 1 at each end of the row, and each interior entry is 1 greater than the sum of the two numbers diagonally above it in the previous row. What is the units digit of the sum of the 2023 numbers in the 2023rd row?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

Problem 21

If A and B are vertices of a polyhedron, define the distance $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if

$\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

(A) $\frac{7}{22}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{1}{2}$

Problem 22

Let f be the unique function defined on the positive integers such that

$$\sum_{d|n} d \cdot f\left(\frac{n}{d}\right) = 1$$

for all positive integers n, where the sum is taken over all positive divisors of n. What is $f(2023)$?

(A) -1536 **(B)** 96 **(C)** 108 **(D)** 116 **(E)** 144

Problem 23

How many ordered pairs of positive real numbers (a, b) satisfy the equation

$$(1 + 2a)(2 + 2b)(2a + b) = 32ab?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** an infinite number

Problem 24

Let K be the number of sequences $A_1, A_2, \cdots, A_n$ such that n is a positive integer less than or equal to 10, each A_i is a subset of $\{1, 2, 3, \cdots, 10\}$,, and A_{i-1} is a subset of A_i for each i between 2 and n, inclusive. For example, $\{\}, \{5, 7\}, \{2, 5, 7\}, \{2, 5, 7\}, \{2, 5, 6, 7, 9\}$ is one such sequence, with $n = 5$.What is the remainder when K is divided by 10?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

Problem 25

There is a unique sequence of integers $a_1, a_2, \cdots, a_{2023}$ such that

$$\tan 2023x = \frac{a_1 \tan x + a_3 \tan^3 x + a_5 \tan^5 x + \cdots + a_{2023} \tan^{2023} x}{1 + a_2 \tan^2 x + a_4 \tan^4 x \cdots + a_{2022} \tan^{2022} x}$$

whenever $\tan 2023x$ is defined. What is a_{2023}?

$\textbf{(A)}\ -2023 \qquad \textbf{(B)}\ -2022 \qquad \textbf{(C)}\ -1 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2023$

Answer Key

1. E
2. A
3. A
4. E
5. B
6. D
7. E
8. D
9. C
10. D
11. C
12. D
13. B
14. E
15. A
16. B
17. E
18. D
19. C
20. C
21. A
22. B
23. B
24. C
25. C