2022 AMC 12B Problems

Problem 1

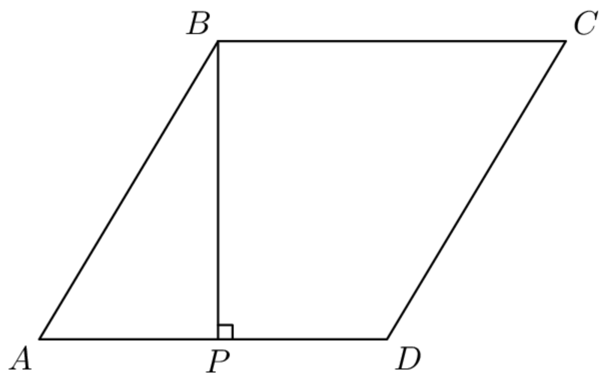
Define $x \diamond y$ to be $|x-y|$ for all real numbers x and y. What is the value of

$$(1 \diamond (2 \diamond 3)) - ((1 \diamond 2) \diamond 3)?$$

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2$

Problem 2

In rhombus $ABCD$, point P lies on segment $\overline{AD}$ such that $BP \perp AD$, $AP = 3$, and $PD = 2$. What is the area of $ABCD$?



$\textbf{(A)}\ 3\sqrt{5} \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 6\sqrt{5} \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Problem 3

How many of the first ten numbers of the sequence 121, 11211, 1112111, ... are prime numbers?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Problem 4

For how many values of the constant k will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 16$

Problem 5

The point $(-1, -2)$ is rotated 270° counterclockwise about the point $(3, 1)$. What are the coordinates of its new position?

$\textbf{(A)}\ (-3,-4) \qquad \textbf{(B)}\ (0,5) \qquad \textbf{(C)}\ (2,-1) \qquad \textbf{(D)}\ (4,3) \qquad \textbf{(E)}\ (6,-3)$

Problem 6

Consider the following 100 sets of 10 elements each:

$$\begin{aligned}&\{1,2,3,\cdots,10\},\\&\{11,12,13,\cdots,20\},\\&\{21,22,23,\cdots,30\},\\&\vdots\\&\{991,992,993,\cdots,1000\}.\end{aligned}$$

How many of these sets contain exactly two multiples of 7?

(A) 40 (B) 42 (C) 43 (D) 49 (E) 50

Problem 7

Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

Problem 8

What is the graph of $y^4 + 1 = x^4 + 2y^2$ in the coordinate plane?

(A) Two intersecting parabolas **(B) Two nonintersecting parabolas**
(C) Two intersecting circles **(D) A circle and a hyperbola**
(E) A circle and two parabolas

Problem 9

The sequence $a_0, a_1, a_2, \cdots$ is a strictly increasing arithmetic sequence of positive integers such that

$$2^{a_7} = 2^{27} \cdot a_7.$$

What is the minimum possible value of a_2?

(A) 8 (B) 12 (C) 16 (D) 17 (E) 22

Problem 10

Regular hexagon $ABCDEF$ has side length 2. Let G be the midpoint of $\overline{AB}$, and let H be the midpoint of $\overline{DE}$. What is the perimeter of $GCHF$?

$\textbf{(A)}\ 4\sqrt{3} \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 4\sqrt{5} \qquad \textbf{(D)}\ 4\sqrt{7} \qquad \textbf{(E)}\ 12$

Problem 11

Let

$$f(n) = \left(\frac{-1 + i\sqrt{3}}{2} \right)^n + \left(\frac{-1 - i\sqrt{3}}{2} \right)^n,$$

where $i = \sqrt{-1}$. What is $f(2022)$?

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 2$

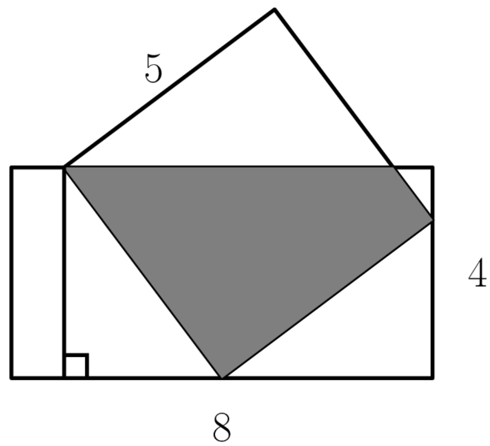
Problem 12

Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2?

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{19}{27} \qquad \textbf{(C)}\ \frac{59}{81} \qquad \textbf{(D)}\ \frac{61}{81} \qquad \textbf{(E)}\ \frac{7}{9}$

Problem 13

The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?



$\textbf{(A)} 15\frac{1}{8}$ $\textbf{(B)} 15\frac{3}{8}$ $\textbf{(C)} 15\frac{1}{2}$ $\textbf{(D)} 15\frac{5}{8}$ $\textbf{(E)} 15\frac{7}{8}$

Problem 14

The graph of

$$y = x^2 + 2x - 15$$

intersects the x-axis at points A and C and the y-axis at point B. What is $\tan(\angle ABC)$?

$\textbf{(A)} \frac{1}{7}$ $\textbf{(B)} \frac{1}{4}$ $\textbf{(C)} \frac{3}{7}$ $\textbf{(D)} \frac{1}{2}$ $\textbf{(E)} \frac{4}{7}$

Problem 15

One of the following numbers is not divisible by any prime number less than 10. Which is it?

(A) $2^{606}-1$ (B) $2^{606}+1$ (C) $2^{607}-1$ (D) $2^{607}+1$ (E) $2^{607}+3^{607}$

Problem 16

Suppose x and y are positive real numbers such that

$$x^y=2^{64} \text{ and } (\log_2 x)^{\log_2 y}=2^{27}.$$

What is the greatest possible value of $\log_2 y$?

$\textbf{(A)}3$ $\textbf{(B)}4$ $\textbf{(C)}3+\sqrt{2}$ $\textbf{(D)}4+\sqrt{3}$ $\textbf{(E)}7$

Problem 17

How many 4×4 arrays whose entries are 0s and 1s are there such that the row sums (the sum of the entries in each row) are $1, 2, 3,$ and $4,$ in some order, and the column sums (the sum of the entries in each column) are also $1, 2, 3,$ and $4,$ in some order? For example, the array

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

satisfies the condition.

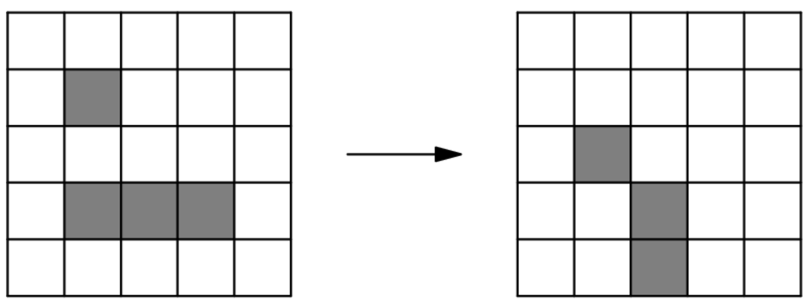
(A) 144 (B) 240 (C) 336 (D) 576 (E) 624

Problem 18

Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

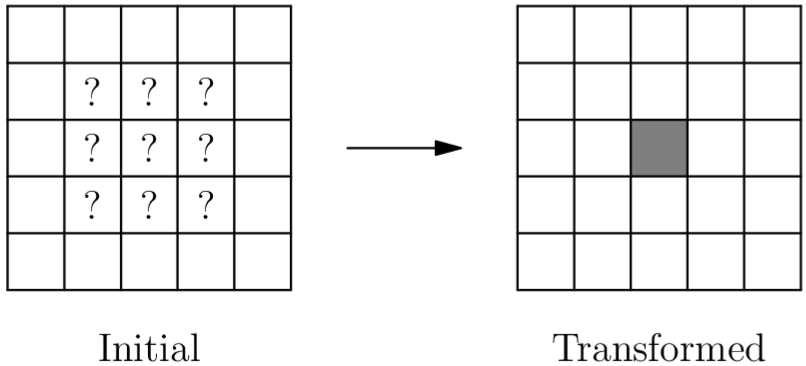
- Any filled square with two or three filled neighbors remains filled.
- Any empty square with exactly three filled neighbors becomes a filled square.
- All other squares remain empty or become empty.

A sample transformation is shown in the figure below.



Initial Transformed

Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)



(A) 14 (B) 18 (C) 22 (D) 26 (E) 30

Problem 19

In $\triangle ABC$ medians $\overline{AD}$ and $\overline{BE}$ intersect at G and $\triangle AGE$ is equilateral. Then $\cos(C)$ can be written as $\frac{m\sqrt{p}}{n}$, where m and n are relatively prime positive integers and p is a positive integer not divisible by the square of any prime. What is $m+n+p$?

(A) 44 (B) 48 (C) 52 (D) 56 (E) 60

Problem 20

Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial x^2+x+1, the remainder is $x+2$, and when $P(x)$ is divided by the polynomial x^2+1, the remainder is $2x+1$. There is a unique polynomial of least degree with these two properties. What is the some of the squares of the coefficients of that polynomial?

$\textbf{(A) } 10 \qquad \textbf{(B) } 13 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 20 \qquad \textbf{(E) } 23$

Problem 21

Let S be the set of circles in the coordinate plane that are tangent to each of the three circles with equations

$$x^2+y^2=4,\ x^2+y^2=64, \text{ and } (x-5)^2+y^2=3.$$

What is the sum of the areas of all circles in S?

$\textbf{(A) } 48\pi \qquad \textbf{(B) } 68\pi \qquad \textbf{(C) } 96\pi \qquad \textbf{(D) } 102\pi \qquad \textbf{(E) } 136\pi$

Problem 22

Ant Amelia starts on the number line at 0 and crawls in the following manner. For $n=1,2,3,$ Amelia chooses a time duration t_n and an increment x_n independently and uniformly at random from the interval $(0,1).$ During the nth step of the process, Amelia moves x_n units in the positive direction, using up t_n minutes. If the total elapsed time has exceeded 1 minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?

$\textbf{(A) } \frac{1}{3} \qquad \textbf{(B) } \frac{1}{2} \qquad \textbf{(C) } \frac{2}{3} \qquad \textbf{(D) } \frac{3}{4} \qquad \textbf{(E) } \frac{5}{6}$

Problem 23

Let x_0, x_1, x_2, $\cdots$ be a sequence of numbers, where each x_k is either 0 or 1. For each positive integer n, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k$$

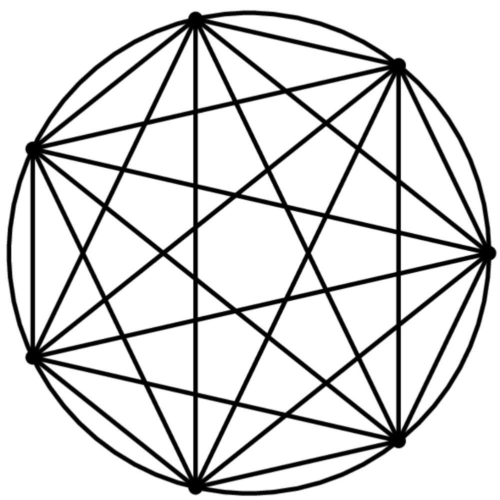
Suppose $7S_n \equiv 1 \pmod{2^n}$ for all $n \geq 1$. What is the value of the sum

$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}?$$

(A) 6 **(B)** 7 **(C)** 12 **(D)** 14 **(E)** 15

Problem 24

The figure below depicts a regular 7-gon inscribed in a unit circle.

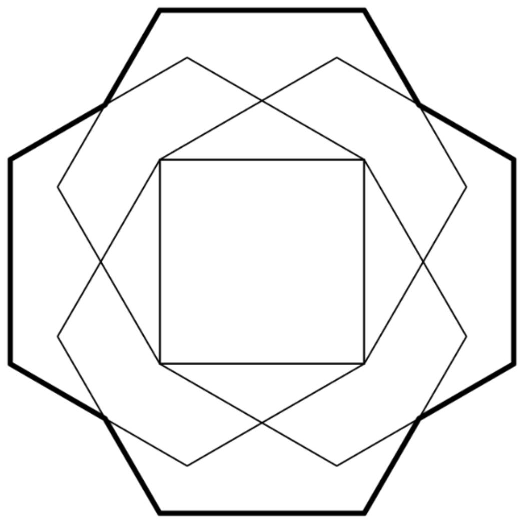


What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?

(A) 49 (B) 98 (C) 147 (D) 168 (E) 196

Problem 25

Four regular hexagons surround a square with a side length 1, each one sharing an edge with the square, as shown in the figure below. The area of the resulting 12-sided outer nonconvex polygon can be written as $m\sqrt{n}+p$, where m, n, and p are integers and n is not divisible by the square of any prime. What is $m+n+p$?



(A) -12 (B) -4 (C) 4 (D) 24 (E) 32

Answer Key

1. A
2. D
3. A
4. B
5. B
6. B
7. D
8. D
9. B
10. D
11. E
12. D
13. D
14. E
15. C
16. C
17. D
18. C
19. A
20. E
21. E
22. C
23. A
24. C
25. B