The Hardest Problems on the 2017 AMC 8 are Extremely Similar to Previous Problems on the AMC 8, 10, 12, Kangaroo, and MathCounts

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We developed a comprehensive, integrated, non-redundant, well-annotated database “CMP” consisting of various competitive math problems, including all previous problems on the AMC 8/10/12, AIME, MATHCOUNTS, Math Kangaroo Contest, Math Olympiads for Elementary and Middle Schools (MOEMS), ARML, USAMTS, Mandelbrot, Math League, Harvard–MIT Mathematics Tournament (HMMT), Princeton University Mathematics Competition (PUMaC), Stanford Math Tournament (SMT), Berkeley Math Tournament (BmMT), the Caltech Harvey Mudd Math Competition (CHMMC), the Carnegie Mellon Informatics and Mathematics Competition (CMIMC). The CPM is an invaluable “big data” system we use for our research and development, and is a golden resource for our students, who are the ultimate beneficiaries.

Based on artificial intelligence (AI), machine learning, and deep learning, we also devised a data mining and predictive analytics tool for math problem similarity searching. Using this powerful tool, we can align query math problems against those present in the target database “CPM,” and then detect those similar problems in the CMP database.
The AMC 8 is a 25-question, 40-minute, multiple choice examination in middle school mathematics designed to promote the development and enhancement of problem solving skills. The problems generally increase in difficulty as the exam progresses. Usually the last 5 problems are the hardest ones.

Among the final 5 problems on the 2017 AMC 8 contest, there is one algebra problem: **Problem 21**; there are 2 discrete math problems (which contains number theory and counting): **Problems 23 and 24**; and there are 2 geometry problems: **Problems 22 and 25**.

For those hardest problems on the 2017 AMC 8, based on the database searching, we found:

- **2017 AMC 8 Problem 21** is almost the same as **1977 AHSME Problem 8**
- **2017 AMC 8 Problem 22** is exactly the same as **Problem 15 on the 2012 International Kangaroo Mathematics Contest -- Junior Level (Class 9 & 10)**, and is very similar to the following 6 problems:
  - **1950 AHSME Problem 35**
  - **1967 AHSME Problem 5**
  - **1970 AHSME Problem 27**
  - **2017 MathCounts State Sprint Problem 24**
  - **2015 MathCounts State Sprint Problem 16**
  - **2012 MathCounts State Sprint Problem 21**
- **2017 AMC 8 Problem 24** is almost the same as the following 2 problems:
  - **2005 AMC 12A Problem 18**
  - **2001 AMC 10 Problem 25/2001 AMC 12 Problem 12**
• **2017 AMC 8 Problem 25** is very similar to the following 4 problems:
  - 2012 AMC 10B Problem 16
  - 2014 AMC 10A Problem 12
  - 2012 AMC 8 Problem 24
  - 1992 AJHSME Problem 24

We can see that **Problem 23** is the only problem that is new and original. Every other problem has strong similarities to previous problems.
Section 1. 2017 AMC 8 Problem 21

2017 AMC 8 Problem 21

Suppose \(a, b,\) and \(c\) are nonzero real numbers, and \(a + b + c = 0.\) What are the possible value(s) for \(\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}?\)

(A) 0 (B) 1 and \(-1\) (C) 2 and \(-2\) (D) 0, 2, and \(-2\) (E) 0, 1, and \(-1\)

This problem is almost the same as 1977 AHSME Problem 8.

1977 AHSME Problem 8

For every triple \((a, b, c)\) of non-zero real numbers, form the number \(\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}.\) The set of all numbers formed is

(A) 0 (B) \(\{-4, 0, 4\}\) (C) \(\{-4, -2, 0, 2, 4\}\) (D) \(\{-4, -2, 2, 4\}\) (E) none of these
Section 2.  2017 AMC 8 Problem 22

2017 AMC 8 Problem 22

In the right triangle $ABC, AC = 12, BC = 5$, and angle $C$ is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?

![Diagram of a right triangle with sides 5, 12, and 13. A semicircle is inscribed within the triangle.]

\[(A) \frac{7}{6} \quad (B) \frac{13}{5} \quad (C) \frac{59}{18} \quad (D) \frac{10}{3} \quad (E) \frac{60}{13}\]

This problem is exactly the same as Problem 15 on the 2012 International Kangaroo Mathematics Contest -- Junior Level (Class 9 & 10), and is very similar to the following 6 problems:

- 1950 AHSME Problem 35
- 1967 AHSME Problem 5
- 1970 AHSME Problem 27
- 2017 MathCounts State Sprint Problem 24
- 2015 MathCounts State Sprint Problem 16
- 2012 MathCounts State Sprint Problem 21

2012 International Kangaroo Mathematics Contest -- Junior Level (Class 9 & 10) Problem 15

The diagram shows a right triangle with sides 5, 12 and 13. What is the radius of the inscribed semicircle?
In triangle $ABC$, $AC = 24$ inches, $BC = 10$ inches, $AB = 26$ inches. The radius of the inscribed circle is:

(A) 26 in  (B) 4 in  (C) 13 in  (D) 8 in  (E) None of these

A triangle is circumscribed about a circle of radius $r$ inches. If the perimeter of the triangle is $P$ inches and the area is $K$ square inches, then $\frac{P}{K}$ is:

(A) independent of the value of $r$  (B) $\frac{\sqrt{2}}{r}$  (C) $\frac{2}{\sqrt{r}}$  (D) $\frac{2}{r}$  (E) $\frac{r}{2}$

In a triangle, the area is numerically equal to the perimeter. What is the radius of the inscribed circle?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6
2017 MathCounts State Sprint Problem 24

What is the radius of the inscribed circle of a triangle with side lengths 9, 13 and 14? Express your answer in simplest radical form.

2015 MathCounts State Sprint Problem 16

What is the radius of a circle inscribed in a triangle with sides of length 5, 12 and 13 units?

2012 MathCounts State Sprint Problem 21

A right triangle has sides with lengths 8 cm, 15 cm and 17 cm. A circle is inscribed in the triangle. In centimeters, what is the radius of the circle?
Section 3.  2017 AMC 8 Problem 24

Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

(A) 78  (B) 80  (C) 144  (D) 146  (E) 152

2017 AMC 8 Problem 24 is equivalent to finding the number of integers among the first 365 positive integers that are not divisible by 3, 4, or 5. This problem is almost the same as the following 2 problems:

- 2005 AMC 12A Problem 18
- 2001 AMC 10 Problem 25/2001 AMC 12 Problem 12

2005 AMC 12A Problem 18

Call a number prime-looking if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

(A) 100  (B) 102  (C) 104  (D) 106  (E) 108

2001 AMC 10 Problem 25/2001 AMC 12 Problem 12

How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

(A) 768  (B) 801  (C) 934  (D) 1067  (E) 1167
Section 4. 2017 AMC 8 Problem 25

2017 AMC 8 Problem 25

In the figure shown, $US$ and $UT$ are line segments each of length 2, and $m\angle TUS = 60^\circ$. Arcs $SR$ and $TR$ are each one-sixth of a circle with radius 2. What is the area of the region shown?

(A) $3\sqrt{3} - \pi$  (B) $4\sqrt{3} - \frac{4\pi}{3}$  (C) $2\sqrt{3}$  (D) $4\sqrt{3} - \frac{2\pi}{3}$  (E) $4 + \frac{4\pi}{3}$

2017 AMC 8 Problem 25 is very similar to the following 4 problems:

- 2012 AMC 10B Problem 16
- 2014 AMC 10A Problem 12
- 2012 AMC 8 Problem 24
- 1992 AJHSME Problem 24

2012 AMC 10B Problem 16

Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?
A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?

(A) $10\pi + 4\sqrt{3}$  (B) $13\pi - \sqrt{3}$  (C) $12\pi + \sqrt{3}$  (D) $10\pi + 9$  (E) $13\pi$

2014 AMC 10A Problem 12

A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?

(A) $27\sqrt{3} - 9\pi$  (B) $27\sqrt{3} - 6\pi$  (C) $54\sqrt{3} - 18\pi$  (D) $54\sqrt{3} - 12\pi$  (E) $108\sqrt{3} - 9\pi$
2012 AMC 8 #24

A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?

\[
(A) \frac{4 - \pi}{\pi} \quad (B) \frac{1}{\pi} \quad (C) \frac{\sqrt{2}}{\pi} \quad (D) \frac{\pi - 1}{\pi} \quad (E) \frac{3}{\pi}
\]

1992 AJHSME Problem 24

Four circles of radius 3 are arranged as shown. Their centers are the vertices of a square. The area of the shaded region is closest to

\[
(A) 7.7 \quad (B) 12.1 \quad (C) 17.2 \quad (D) 18 \quad (E) 27
\]
Section 5. Conclusions

This year’s AMC 8 was more difficult than the last year’s AMC 8. Some hard problems were even AMC 10 level. For example, Problem 23 and Problem 24 on the 2017 AMC 8 are two typical AMC 10 hard problems.

Problem 23 is involved in detecting a sequence of four factors of 60 that forms an arithmetic progression with a common difference of 5.

Problem 24 is equivalent to finding the number of integers among the first 365 positive integers that are not divisible by 3, 4, or 5. We should use the principle of inclusion and exclusion (for 3 sets) to solve this problem.

Because the AMC 8 problems are getting harder, we must practice not only previous AMC 8 problems but also easy, medium, and even high difficulty level problems from previous AMC 10 to do well on the AMC 8.