Mock Examination

AMC 12
American Mathematics Contest 12

Test Sample

Make time to take the practice test.
It’s one of the best ways to get ready for the AMC.
INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. **SCORING:** You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, **sign your name** in the space provided on the Answer Form.
AMC 12 Mock Test Problems

Problem 1
Several three-legged tables and four-legged tables have a total of 23 legs. If there is more than one table of each type, what is the number of three-legged tables?
(A) 1  (B) 2  (C) 3  (D) 4  (E) 5

Problem 2
If \((x - y)^2 = 169\) and \((x + y)^2 = 225\), what is \(xy\)?
(A) 13  (B) 14  (C) 15  (D) 4  (E) 56

Problem 3
The two semicircles in the figure below have centers \(R\) and \(S\), respectively. If \(RS = 17\), what is the total length of the darkened curve?

(A) 11\(\pi\)  (B) 12\(\pi\)  (C) 13\(\pi\)  (D) 15\(\pi\)  (E) 17\(\pi\)

Problem 4
If
\[
\frac{a}{b + c + d} = \frac{4}{3} \quad \text{and} \quad \frac{a}{b + c} = \frac{3}{5},
\]
then the value of \(\frac{d}{a}\) is

https://ivyleaguecenter.wordpress.com/
Problem 5

If \( x_1, x_2, \) and \( x_3 \) are the roots of the equation \( x^3 - 2x + 6 = 0 \), what is the numerical value of \((x_1 + 1)(x_2 + 1)(x_3 + 1)\)?

(A) \(-7\)  (B) \(-4\)  (C) \(4\)  (D) \(5\)  (E) \(8\)

Problem 6

Before Brad started a five-hour drive, his car's odometer reading was 38983, a palindrome. (A palindrome is a number which reads the same when read forward as it does when read backward). At his destination, the odometer reading was another palindrome. If Brad never exceed the speed limit of 70 miles per hour, which of the following was his maximum possible average speed?

(A) 22  (B) 33  (C) 52.5  (D) 62  (E) 65

Problem 7

If \( f(x) = x^{2017} + 2 \) and \( g(x) = \sqrt[2017]{x - 2} \), then \( f^{-1} \circ g^{-1}(2017) \) is

(A) \(-2\)  (B) \(2\)  (C) 2015  (D) 2017  (E) 2019

Problem 8

To shovel all of the snow on his driveway, Alex needs 12 hours. Individually, Bob needs 8 hours to shovel all of Alex’s snow, Carl needs 6 hours to shovel all of Alex’s snow, and Allison needs 4 hours to shovel all of Alex’s snow. If Alex, Bob, Carl, and Dick all work together, how many minutes do they need to shovel all of Alex’s snow?

(A) 120  (B) 108  (C) 96  (D) 90  (E) 84
Problem 9
If
\[8 \sin \alpha + 15 \cos \alpha = 17,\]
then \(\tan \alpha\) is
(A) 1       (B) –1       (C) \(\frac{8}{15}\)       (D) \(\frac{15}{8}\)       (E) 0

Problem 10
Michael wants to fill his swimming pool using two hoses, each of which sprays water at a constant rate. Hose X fills the pool in \(x\) hours when used by itself, where \(x\) is a positive integer. Hose Y fills the pool in \(y\) hours when used by itself, where \(y\) is a positive integer. When used together, Hose X and Hose Y fill the pool in 10 hours. How many different possible values are there for \(y\)?
(A) 9       (B) 10       (C) 11       (D) 12       (E) 15

Problem 11
How many integers \(x\) satisfy the equation \((x^2 - 5x + 5)^{x^2-9x+20} = 1\)?
(A) 2       (B) 3       (C) 4       (D) 5       (E) 6

Problem 12
Let \(f(x)\) be a real function such that \(f(x+y) = f(xy)\) for all real numbers \(x\) and \(y\), and \(f(8) = 5\). Find the value of \(f(2017^{2017})\).
(A) 1       (B) 5       (C) 8       (D) 40       (E) 2014

Problem 13
Let \(w, x, y,\) and \(z\) be integers satisfying
\[w \log_{10} 2 + x \log_{10} 3 + y \log_{10} 5 + z \log_{10} 7 = 2017.\]
What is \( w + x + y + z \)?

(A) 2016    (B) 2017    (C) 4034    (D) 5012    (E) 5034

**Problem 14**

A rectangular piece of paper measures 17 inches by 8 inches. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area in square inches of the new figure?

\[ \text{Before} \quad \text{After} \]

(A) 64    (B) 72    (C) 81    (D) 104    (E) 168

**Problem 15**

Let

\[ f(k) = \left( \frac{1 + i}{\sqrt{2}} \right)^k + \left( \frac{1 - i}{\sqrt{2}} \right)^k, \]

where \( i^2 = -1 \). Determine the value of \( f(2005) + f(2017) \).

(A) 0    (B) \( \frac{2i}{\sqrt{2}} \)    (C) \( i \)    (D) \( \frac{2}{\sqrt{2}} \)    (E) \( -\frac{2i}{\sqrt{2}} \)

**Problem 16**

If \( \alpha = 15^\circ \) and \( \beta = 30^\circ \), then the value of \((1 + \tan \alpha)(1 + \tan \beta)\) is
Problem 17

Two points are picked at random on the unit circle $x^2 + y^2 = 1$. What is the probability that the chord joining the two points has length at least 1?

(A) $\frac{1}{4}$  (B) $\frac{1}{3}$  (C) $\frac{1}{2}$  (D) $\frac{2}{3}$  (E) $\frac{3}{4}$

Problem 18

A millipede has one sock and one shoe for each of its $n$ legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

(A) $n!$  (B) $2^n \cdot n!$  (C) $(n!)^2$  (D) $\frac{(2n)!}{2^n}$  (E) $(2n)!$

Problem 19

A rectangular piece of paper, $PQRS$, has $PQ = 20$ and $QR = 15$. The piece of paper is glued at on the surface of a large cube so that $Q$ and $S$ are at vertices of the cube. (Note that $\triangle QPS$ and $\triangle QRS$ lie at on the front and top faces of the cube, respectively.) The shortest distance from $P$ to $R$, as measured through the cube, is
Problem 20

Alan has a number of gold bars, all of different weights. He gives the 24 lightest bars, which weigh 45% of the total weight, to Bob. He gives the 13 heaviest bars, which weigh 26% of the total weight, to Carl. She gives the rest of the bars to Dale. How many bars did Dale receive?

(A) 19   (B) 18   (C) 17   (D) 16   (E) 15

Problem 21

A sphere is divided into regions by 100 planes that are passing through its center. What is the maximum possible number of regions that are created on its surface?

(A) \(2^{100}\)   (B) \(2^{99}\)   (C) \(10000\)   (D) \(9902\)   (E) \(9900\)

Problem 22

A standard die is rolled until a six rolls. Each time a six does not roll, a fair coin is tossed and a running tally of the number of heads minus the number of tails tossed is kept. For example, if the die rolls are 5, 2, 1, 6 and the tosses are H, H, T, then the running tally is 1, 2, 1. What is the probability that the absolute value of this running tally never equals 3?

(A) \(\frac{289}{444}\)   (B) \(\frac{7}{10}\)   (C) \(\frac{81}{112}\)   (D) \(\frac{3}{4}\)   (E) \(\frac{5}{6}\)

Problem 23

A coin that is 8 units in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 units. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it
is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

\[
\begin{array}{c}
\text{win} \\
\text{lose} \\
\text{lose} \\
\text{lose} \\
\end{array}
\]

(A) \( \frac{1}{49} \)  
(B) \( \frac{5}{147} \)  
(C) \( \frac{7}{25} \)  
(D) \( \frac{25}{441} \)  
(E) \( \frac{4\pi}{25} \)

**Problem 24**

Let \( x, y, \) and \( z \) be positive real numbers such that

\[
x + y + z = 1 \quad \text{and} \quad xy + yz + zx = \frac{1}{3}.
\]

The number of possible values of the expression

\[
\frac{x}{y} + \frac{y}{z} + \frac{z}{x}
\]

is

(A) 1  
(B) 2  
(C) 3  
(D) More than 3 but finitely many  
(E) Infinitely many

**Problem 25**
James places a counter at 0 on the diagram. On his first move, he moves the counter 11 step clockwise to 1. On his second move, he moves 22 steps clockwise to 5. On his third move, he moves 33 steps clockwise to 2. He continues in this manner, moving \( n^n \) steps clockwise on his \( n \)th move. At which position will the counter be after 1234 moves?

(A) 1   (B) 3   (C) 5   (D) 6   (E) 7
Answer Key

1. E
2. B
3. E
4. D
5. A
6. D
7. D
8. C
9. C
10. A
11. D
12. B
13. C
14. D
15. A
16. B
17. D
18. D
19. B
20. E
21. D
22. A
23. D
24. A
25. E