AMC 12 Mock Test Problems

Problem 1

A bag contains 8 red balls, a number of white balls, and no other balls. If $5/6$ of the balls in the bag are white, then the number of white balls in the bag is

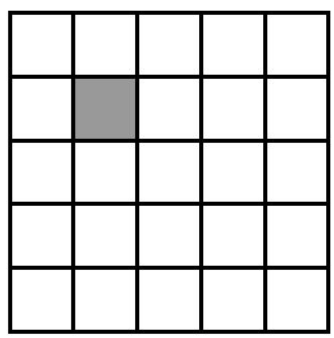
(A) 48 (B) 46 (C) 40 (D) 32 (E) 30

Problem 2

Let A and B be two distinct points in the xy-plane. In how many different places in the xy-plane can a third point, C, be placed such that $AB = BC = CA$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 3



Given a 5×5 grid, how many of squares contain the shaded unit square?

(A) 8 (B) 10 (C) 11 (D) 12 (E) 14

Problem 4

A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive in increasing order?

(A) 458 (B) 424 (C) 376 (D) 315 (E) 288

Problem 5

The line with equation $y = 2x$ is translated 2 units to the left and 4 units up. What is the x-intercept of the resulting line?

(A) -4 (B) -2 (C) 2 (D) 4 (E) 6

Problem 6

The faces of each of three standard fair 5-sided dice are numbered 2, 4, 6, 8, and 10. When the three dice are tossed, what is the probability that their product will *not* be a power of 2?

(A) $\frac{2}{5}$ (B) $\frac{64}{125}$ (C) $\frac{3}{5}$ (D) $\frac{98}{125}$ (E) $\frac{117}{125}$

Problem 7

Let $P(x) = x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$ be a polynomial with real coefficients. If $1 - i$ and i are two roots of $P(x)$, what is the value of $a_1 + a_2 + a_3 + a_4$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Problem 8

Square $ABCD$ has side length 1. Points M and N are the midpoints of AB and BC, respectively. Find the value of $\sin \angle MDN$.

(A) $\frac{1}{3}$ (B) $\frac{3}{5}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{5}}{3}$ (E) $\frac{\sqrt{3}}{2}$

Problem 9

Let a and b be positive integers such that

$$\sqrt{7 + \sqrt{48}} = a + \sqrt{b}.$$

Then the value of ab is

(A) 28 (B) 21 (C) 14 (D) 8 (E) 6

Problem 10

Let m and n be the numbers of digits in 4^{2019} and 25^{2019}, respectively. What is the value of $m+n$?

(A) 4038 (B) 4039 (C) 4040 (D) 4041 (E) 4042

Problem 11

A closed right circular cylinder has an integer radius and an integer height. The numerical value of its volume is ten times the numerical value of its surface area. How many distinct cylinders satisfy this property?

(A) 8 (B) 10 (C) 12 (D) 15 (E) 20

Problem 12

Find the probability that a randomly chosen positive divisor of 6^{19} is an integer multiple of 6^{10}.

(A) $\frac{1}{4}$ (B) $\frac{9}{19}$ (C) $\frac{1}{2}$ (D) $\frac{10}{19}$ (E) $\frac{11}{20}$

Problem 13

Alan and Ben ran a race. They ran at a constant speed throughout the race. Alan began the race 31 meters ahead of Ben. After 3 minutes, Ben was 20 meters ahead of Alan. Ben won the race exactly 7 minutes after it began. How far in meters from the finish line was Alan when Ben won?

(A) 20 (B) 51 (C) 64 (D) 88 (E) 94

Problem 14

For how many positive integers x is

$$(x-1)(x-3)(x-5)\cdots(x-2017)(x-2019)<0\,?$$

(A) 504 (B) 505 (C) 1008 (D) 1009 (E) 1010

Problem 15

Which of the following polynomials has root:

$$x = \sqrt[3]{5+\sqrt{17}} + \sqrt[3]{5-\sqrt{17}}\,?$$

(A) $x^3 - 6x - 10$

(B) $x^3 - 6x + 10$

(C) $x^3 - 6x^2 - 10$

(D) $x^3 - 6x^2 + 10$

(E) $x^3 + 6x^2 + 6x + 10$

Problem 16

The parabola with equation $y = x^2$ intersects the line with equation $y = 3tx + 4t^2$ at points A and B, where $t > 0$. Let O be the origin. If the area of ΔOAB is 640, then what is the value of t?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 17

If a and b are integers such that

$$\frac{100!}{225^a \cdot 49^b}$$

is an integer, what is the maximum possible value of $a + b$?

(A) 10 (B) 16 (C) 20 (D) 24 (E) 33

Problem 18

Let n denote the number of ordered pairs (x, y) of integers such that

$$\log_x y + 2\log_y x = 3,$$

where $2 \le x \le 2020$ and $2 \le y \le 2020$. Then what is the product of the digits of n?

(A) 14 (B) 13 (C) 12 (D) 11 (E) 10

Problem 19

Given that

$$(1+\sin x)(1+\cos x) = \frac{5}{4}$$

and

$$(1-\sin x)(1-\cos x) = \frac{a}{b} - \sqrt{c},$$

where $a, b,$ and c are positive integers with a and b relatively prime, find the value of $a+b+c$.

(A) 12 (B) 16 (C) 21 (D) 27 (E) 33

Problem 20

How many integer triples (a, b, c) are there such that

$$(x-a)(x-b) = (x+c)(x-6)+5$$

for all real numbers x?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 21

Let $\{t_n\}$ be a sequence with the first term is $t_1 = a$ and the third term $t_3 = b$. The terms of the sequence have the property that every term after the first term equals 1 less than the sum of the terms immediately before and after it. What is the sum of the first 2019 terms in the sequence?

(A) $-a-2b+2025$ (B) $3a-2b+2017$ (C) b
(D) $a+b-1$ (E) $2a+2b+2015$

Problem 22

The sets $A=\{z: z^{18}=1\}$ and $B=\{w: w^{48}=1\}$ are both sets of complex roots of unity. The set $C=\{zw: z\in A \text{ and } w\in B\}$ is also a set of complex roots of unity. Let N be the number of the distinct elements are in C. What is the product of the digits of N?

(A) 12 (B) 16 (C) 20 (D) 32 (E) 42

Problem 23

A container in the shape of a triangular prism stands on one of its triangular faces. Three mutually tangent spheres of radius 1 are placed inside the container, each touching the triangular bottom. Each sphere touches two of the rectangular faces of the container. A fourth sphere of radius 1 rests on the three spheres, touching each of the three spheres and the top of the prism. What is the volume of the prism?

(A) $10+8\sqrt{2}+8\sqrt{3}+3\sqrt{6}$ (B) $10+8\sqrt{2}+8\sqrt{3}+4\sqrt{6}$

(C) $12+8\sqrt{2}+8\sqrt{3}+4\sqrt{6}$ (D) $12+9\sqrt{2}+8\sqrt{3}+4\sqrt{6}$

(E) $12+8\sqrt{2}+8\sqrt{3}+5\sqrt{6}$

Problem 24

A standard fair six-sided die is rolled four times. The probability that each of the last three rolls is at least as large as the roll preceding it can be written as a fraction $\frac{a}{b}$ in lowest terms. What is the value of $b-a$?

(A) 21 (B) 32 (C) 43 (D) 54 (E) 65

Problem 25

For a positive integer n and nonzero digits x, y, and z, let X_n be the n-digit integer each of whose digits is equal to x; let Y_n be the n-digit integer each of whose digits is equal to y, and let Z_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to z. What is the number of quadruples (n, X_n, Y_n, Z_n) for which $1 \leq n \leq 2019$, and

$$Z_n - Y_n = X_n^2 ?$$

(A) 2019 (B) 4034 (C) 4038 (D) 4042 (E) 4045

Answer Key

1. B
2. B
3. E
4. A
5. A
6. D
7. C
8. B
9. E
10. B
11. D
12. A
13. D
14. B
15. A
16. D
17. C
18. E
19. D
20. D
21. E
22. B
23. C
24. E
25. E