14 Problems Appeared on Both the 2016AMC 10A and 12A

1. 2016 AMC 10A #1/2016 AMC 12A #1

What is the value of

$$\frac{11! - 10!}{9!}?$$

$\textbf{(A)}\ 99 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 110 \qquad \textbf{(D)}\ 121 \qquad \textbf{(E)}\ 132$

2. 2016 AMC 10A #2/2016 AMC 12A #2

For what value of x does $10^x \cdot 100^{2x} = 1000^5$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

3. 2016 AMC 10A #4/2016 AMC 12A #3

The remainder can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x, y) = x - y\left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$?

$\textbf{(A)}\ -\frac{3}{8} \qquad \textbf{(B)}\ -\frac{1}{40} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \frac{3}{8} \qquad \textbf{(E)}\ \frac{31}{40}$

4. 2016 AMC 10A #7/2016 AMC 12A #4

The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

(A) 50 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 100

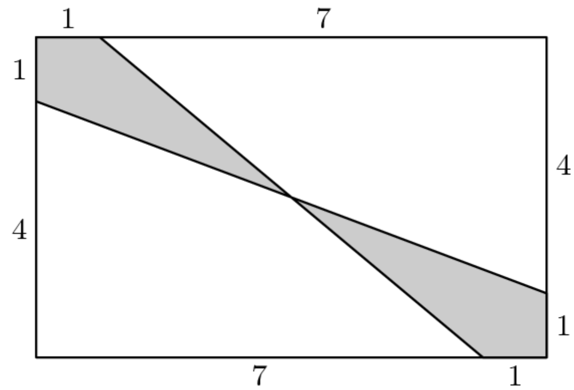
5. 2016 AMC 10A #9/2016 AMC 12A #6

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

6. 2016 AMC 10A #11/2016 AMC 12A #8

What is the area of the shaded region of the given 8×5 rectangle?



(A) $4\frac{3}{5}$ **(B)** 5 **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{2}$ **(E)** 8

7. 2016 AMC 10A #13/2016 AMC 12A #10

Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

8. 2016 AMC 10A #17/2016 AMC 12A #13

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 20$

9. 2016 AMC 10A #18/2016 AMC 12A #14

Each vertex of a cube is to be labeled with an integer 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 24$

10. 2016 AMC 10A #21/2016 AMC 12A #15

Circles with centers P, Q, and R, having radii 1, 2, and 3, respectively, lie on the same side of line l and are tangent to l at P', Q', and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \sqrt{\frac{2}{3}} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \sqrt{6}-\sqrt{2} \qquad \textbf{(E)}\ \sqrt{\frac{3}{2}}$

11. 2016 AMC 10A #22/2016 AMC 12A #18

For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

$\textbf{(A)}\ 110 \qquad \textbf{(B)}\ 191 \qquad \textbf{(C)}\ 261 \qquad \textbf{(D)}\ 325 \qquad \textbf{(E)}\ 425$

12. 2016 AMC 10A #23/2016 AMC 12A #20

A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers a, b, and c. (Here the dot $\cdot$ represents multiplication). The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

$\textbf{(A)}\ 109 \qquad \textbf{(B)}\ 201 \qquad \textbf{(C)}\ 301 \qquad \textbf{(D)}\ 3049 \qquad \textbf{(E)}\ 33,601$

13. 2016 AMC 10A #24/2016 AMC 12A #21

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of the fourth side?

$\textbf{(A)}\ 200 \qquad \textbf{(B)}\ 200\sqrt{2} \qquad \textbf{(C)}\ 200\sqrt{3} \qquad \textbf{(D)}\ 300\sqrt{2} \qquad \textbf{(E)}\ 500$

14. 2016 AMC 10A #25/2016 AMC 12A #22

How many ordered triples (x, y, z) of positive integers satisfy $\operatorname{lcm}(x, y) = 72, \operatorname{lcm}(x, z) = 600$, and $\operatorname{lcm}(y, z) = 900$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 64$