Some Problems on the 2016 AMC 10/12 are Exactly the Same as Previous AMC/ARML Problems

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We have developed a comprehensive, integrated, non-redundant, well-annotated database "**CPM**" consisting of competitive math problems, including all previous AMC 10/12 problems, AIME problems, ARML problems, HMMT problems, Math League problems, PUMaC problems, Stanford Math Tournament (SMT) problems. The CPM is an invaluable **"big data" system we use for our research, and is a golden resource for our students, who are the ultimate beneficiaries.**

We have also devised a **data mining and predictive analytics tool** for *math problem similarity searching*. Using this powerful tool, we can align query math problems against those present in the target database "CPM," and then find those similar problems in the CMP database.

For the 2016 AMC10/12A and B problems, based on the database searching, we have found:

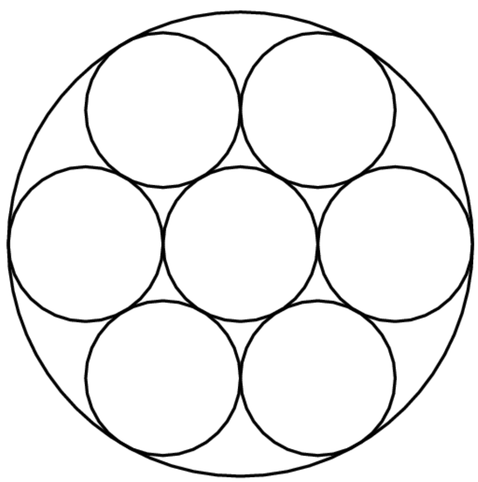
- **2016 AMC 10A Problem 15 is similar to 2002 AMC 10A #5**
- **2016 AMC 10A Problem 18 is similar to 2007 AMC 10A #11.**
- **2016 AMC 10B Problem 21 is completely the *same* as 2014 ARML Team Round Problem 8**
- **2016 AMC 10B Problem 21 is similar to the following problems:**
    - 2008 AMC 12A Problem 14
    - 1987 AIME Problem 4
    - 2014 University of Maryland High School Mathematics Competition Problem 16

In my AMC 10/12 Prep Class on Feb. 14, 2016, I used Problem 8 in the 2014 ARML Team Round and the 2008 AMC 12A Problem 14, as two typical examples**,** to illustrate how to efficiently compute the area of the region defined by inequalities or bounded by a simple closed curve. Thus, when my students attended the 2016 AMC 10B, they already knew how to solve this exact problem and its answer: $\mathbf{2+\pi}$. So they took one second to bubble the correct answer (B) and then got 6 points easily!

1. 2016 AMC 10A Problem 15 is similar to 2002 AMC 10A #5

2016 AMC 10A #15

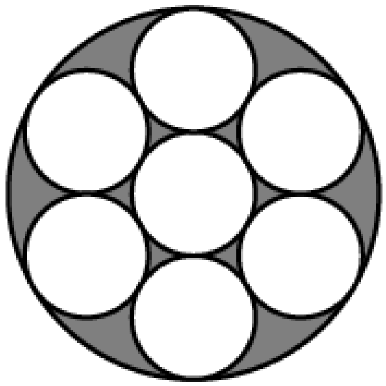
Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



(A) $\sqrt{2}$ **(B)** 1.5 **(C)** $\sqrt{\pi}$ **(D)** $\sqrt{2\pi}$ **(E)** π

2002 AMC 10A #5/2002 AMC 12A #5

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



(A) π (B) 1.5π (C) 2π (D) 3π (E) 3.5π

The 2002 AMC 10A #5 is regarding the area of the shaded region. Furthermore, the 2016 AMC 10A #15 is concerning the radius of the shaded region.

2. 2016 AMC 10A Problem 18 is similar to 2007 AMC 10A #11

2016 AMC 10A Problem 18 is the hardest problem on the contest. The original idea comes from 2007 AMC 10A #11. 2016 AMC 10A Problem 18 is an extension of 2007 AMC 10A #11.

2016 AMC 10A #18

Each vertex of a cube is to be labeled with an integer 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

(A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

2007 AMC 10A #11

The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 24

3. 2016 AMC 10B Problem 21 is completely same as 2014 ARML Team Round Problem 8

2016 AMC 10B Problem 21

What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

(A) $\pi + \sqrt{2}$ **(B)** $\pi + 2$ **(C)** $\pi + 2\sqrt{2}$ **(D)** $2\pi + \sqrt{2}$ **(E)** $2\pi + 2\sqrt{2}$

2014 ARML Team Round Problem 8

Compute the area of the region defined by $x^2 + y^2 \leq |x| + |y|$.

The 2016 AMC 10B Problem 21 is exactly the same as the 2014 ARML Team Round Problem 8. However, the mathematical description in **2016 AMC 10B Problem 21 is WRONG**. In Euclidean geometry, the area of a region enclosed by a curve must be bound by a *closed simple curve*. In the plane, a *closed curve* is a curve that forms a path whose starting point is also its ending point — that is, a path from any of its points to the same point. A curve is *simple* if it does not cross itself. A *plane bound by a closed simple curve* is actually defined as a non-self-intersecting continuous loop in the plane. Typical examples are circles, ellipses, and polygons.

However, in the **2016 AMC 10B Problem 21,** the equation $x^2 + y^2 = |x| + |y|$ gives four circles:

$$\left(x \pm \frac{1}{2}\right)^2 + \left(y \pm \frac{1}{2}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

and they intersect each other. Thus, the graph of the equation $x^2 + y^2 = |x| + |y|$ is self-intersecting, which is not a simple closed curve. The area of a region enclosed by such a non-simple curve is *not well-defined*. Hence, it does not satisfy the conditions necessary for Euclidean geometry, and should not be on the test as it is not defined.

The 2016 AMC 10/12 test makers completely copied the 2014 ARML Team Round Problem 8 for the 2016 AMC 10B Problem 21. However, they did not accurately replicate it. The original problem on the 2014 ARML is mathematically accurate but the AMC 10B Problem 21 has an error that disproves its validity.

Solution of 2014 ARML Team Round Problem 8

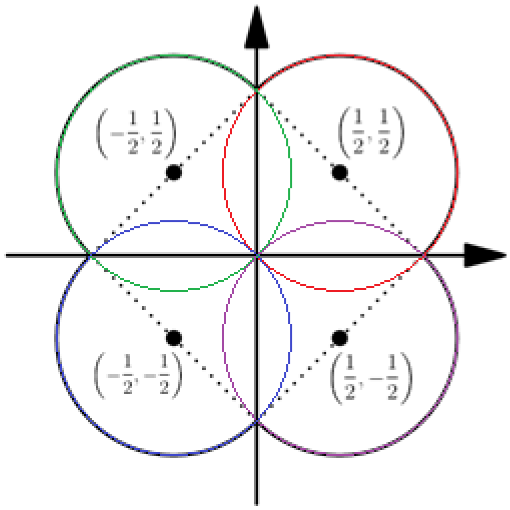
The equation $x^2 + y^2 = |x| + |y|$ gives 4 circles:

$$x^2 + y^2 \pm x \pm y = 0,$$

which are equivalent to:

$$\left(x \pm \frac{1}{2}\right)^2 + \left(y \pm \frac{1}{2}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2.$$

The region defined by $x^2 + y^2 \leq |x| + |y|$ is actually the union of 4 disks bounded by the above 4 circles, respectively, each with radius $\frac{\sqrt{2}}{2}$, as shown below.



Note that the region is symmetric in all 4 quadrants, so it suffices to find the area of the portion in the first quadrant. This portion consists of a right isosceles triangle with leg lengths of 1 and a semicircle of radius $\frac{\sqrt{2}}{2}$. Its area is: $\frac{1}{2}+\frac{\pi}{4}$. Thus, the area of the region is:

$$4\left(\frac{1}{2}+\frac{\pi}{4}\right)=2+\pi.$$

4. 2016 AMC 10B Problem 21 is similar to three problems from previous math contests

2016 AMC 10B Problem 21 is similar to the following problems:

- 2008 AMC 12A Problem 14
- 1987 AIME Problem 4
- 2014 University of Maryland High School Mathematics Competition Problem 16

2008 AMC 12A Problem 14

What is the area of the region defined by the inequality $|3x-18|+|2y+7|\leq 3$?

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 5

Solution

Because area is invariant under translation, we know that after translating left 6 and up $\frac{7}{2}$ units, the inequality is changed into

$$|3x| + |2y| \leq 3,$$

which forms a rhombus centered at the origin and vertices at $(\pm 1, 0), (0, \pm 1.5)$. Thus, the diagonals are of length 2 and 3. Recall that the area is half the product of the diagonals. The answer is (A): $\frac{2\times 3}{2} = 3$.

1987 AIME Problem 4

Find the area of the region enclosed by the graph of $|x-60| + |y| = \left|\frac{x}{4}\right|$.

Solution 1

The equation $|y| = \left|\frac{x}{4}\right| - |x-60|$ gives 4 lines:

$$y = \pm\frac{1}{4}x \pm (x-60).$$

That is:

$$y = \frac{5}{4}x - 60,$$

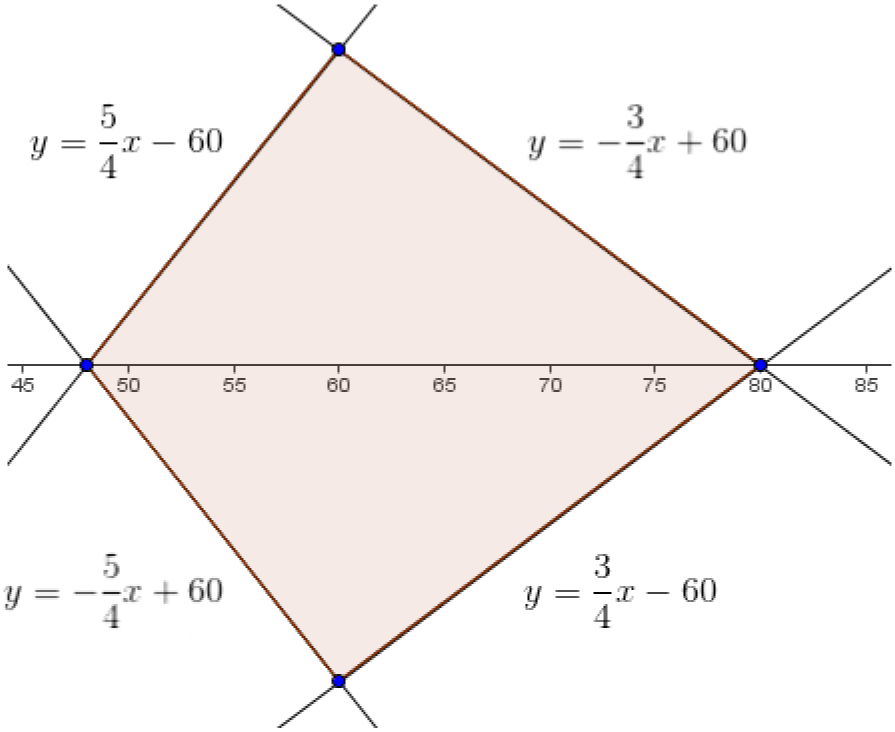
$$y = -\frac{3}{4}x + 60,$$

$$y = \frac{3}{4}x - 60,$$

$$y = -\frac{5}{4}x + 60.$$

These four lines form a kite whose vertices are the intersections of the lines:

$$(48, 0), (60, 15), (80, 0), (60, -15).$$

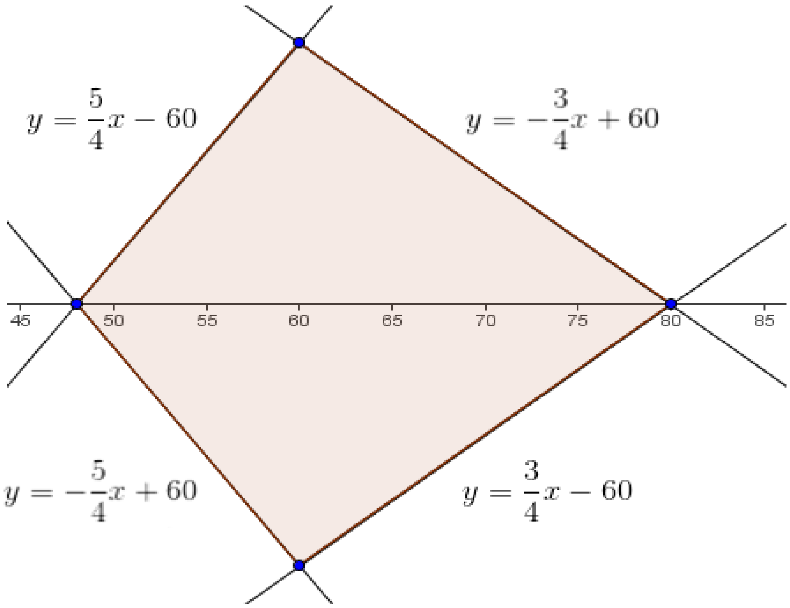


Thus, the lengths of the diagonals of the kite are 32 and 30. Note that the area of a kite is half the product of the diagonals. Hence, the area of the region is:

$$\frac{32 \times 30}{2} = 480.$$

Solution 2

The graph is a quadrilateral, so we need the four vertices to find its area.



Since $|y|$ is nonnegative, $\left|\frac{x}{4}\right| \geq |x-60|$. Solving this gives two inequalities:

$$\frac{x}{4} \geq x - 60, \quad \frac{x}{4} \geq -(x-60).$$

Thus, $48 \leq x \leq 80$. Note that y achieves its maximum value 15 and minimum value -15 when $|x-60| = 0$, i.e., $x = 60$, yielding the vertices $(60, \pm 15)$. Since the graph is symmetric about the y-axis, we just need casework upon x. Since $\frac{x}{4} > 0$, we break up the condition $|x-60|$:

- if $x - 60 > 0$, then $y = -\frac{3}{4}x + 60$,
- if $x - 60 < 0$, then $y = \frac{5}{4}x + 60$.

The area of the region enclosed by the graph is that of the quadrilateral with four vertices: $(48, 0), (60, 15), (80, 0), (60, -15)$. Breaking it up into triangles and solving, we obtain:

$$2 \cdot \frac{1}{2}(80 - 48)15 = 480.$$

2014 University of Maryland High School Mathematics Competition Problem 16

Let S denote the set of points $(x, \ y)$ in the plane that satisfy the inequality $|x-3| + |y-6| = 10$.The area of the region S is

(A) 50 (B) 100 (C) 150 (D) 200 (E) 400

Solution

Note that area is invariant under translation. Translating left 3 and down 6 units, the inequality is changed into

$$|x| + |y| \leq 10,$$

which forms a square with a diagonal of 20, centered at the origin and vertices at $(\pm 10, 0), (0, \pm 10)$. Recall that the area is half the product of the diagonals. The answer is (D):

$$\frac{20 \times 20}{2} = 200.$$

Conclusions

All problems from past AMC 10/12 exams (2000-2016) and AHSME (1950–1999) form our "big data" system. The AHSME (American High School Mathematics Examination) was the former name of the AMC, before 2000. **We have used data mining and predictive analytics to extensively examine the types and the frequencies of questions in all these materials, and then completely "decoded" the AMC 10/12.** We always show all the "secret code" cracked from the above big data to students, and teach them to totally grasp and "control" the AMC. **For all questions on the recent AMC contests, we can find their "ancestors" and "roots" from the old AMC problems.** Therefore, ***the best way to prepare for the contest is to practice by solving old AMC problems.***

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