Middle School Competitive Math Course

Homework Sample

Developed by Dr. Henry Wan

Homework: On average, **at least 2 hours per week**.

We will carefully review and check their homework, and correct any mistakes. Based on his/her work, we will provide the student with individualized proposal and support.

The only way to learn mathematics is by doing mathematics! Homework assignments are a fundamental part of a mathematics course. Homework is also seen as a welcome challenge and an opportunity for further learning. Homework given prior to a lesson can aid in understanding later during class. Homework also provides opportunities for reinforcement of the material learned in class.

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The following steps are important for solving math problems:

FIND OUT

- **Read** the **problem twice**.

- Underline or **highlight key words** and numerical data.

- Decide what the **question** is asking for.

- **You may need to** translate expressions and equations that are written using English words/sentences into mathematical symbols and operations.

- Have you seen a similar problem before?

- If so, how is this problem similar? How is it different?

- What facts do you have?

- **What conditions are given?**

- What do you know that is not stated in the problem?

CHOOSE A STRATEGY

- How have you solved similar problems in the past?

- What strategies do you know?

- Try a strategy that seems as if it will work.

- If it doesn't, it may lead you to one that will.

SOLVE IT

- Use the *strategy* you selected and work the problem.

LOOK BACK

- Reread the question.

- Did you answer the question asked?

- Is your answer in the correct units?

- Does your answer seem reasonable?

Specific *strategies* may vary in name. Most, however, fall into these basic categories:

- Compute or Simplify (C)

- Use a Formula (F)

- Make a Model or Diagram (M)

- Make a Table, Chart or List (T)

- Guess, Check & Revise (G)

- Consider a Simpler Case (S)

- Eliminate (E)

- Look for Patterns (P)

The "Show steps" feature allows you to learn basic mathematics on your own, or it can simply be a nice way to check your work!

Mathematics is fun and beautiful. It is an art.

- Try to solve many different types of problems with the same method.
- Try also to solve one problem with different methods.

Name: _____   **Date:** _____

The weekly homework problem sets consist of three types of problems:

(1) **A fast/mental calculation problem set**: this will help students establish excellent basic calculation skills and fast problem-solving ability making all areas of math easier and quicker.

(2) **A comprehensive problem set**: this set consists of 25 questions at the AMC 8/MathCounts level taken from the licensed AMC Database. The set covers the following areas: Counting, Number Theory, Logic Reasoning, Probability, Statistics, Algebra, and Geometry.

(3) **An especially designed problem set**: this focuses on the topic presented in an immediately previous lesson, and provides students with the needed review and reinforcement about material learned in class.

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Please solve as many problems as you can. You must show all your work on this sheet.

Part I:

Fast/Mental Calculation Problem Set

Use the efficient strategy, based on the formula $(a + b)(a - b) = a^2 - b^2$, we introduced in the immediately previous class to solve the following problems.

Example:

$$113 \times 109 = (111 + 2)(111 - 2) = 111^2 - 2^2 = 12321 - 4 = 12317$$

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1. $14 \times 12 =$ _____

2. $19 \times 13 =$ _____

3. $34 \times 46 =$ _____

4. $73 \times 87 =$ _____

5. $104 \times 116 =$ _____

6. $113 \times 109 =$ _____

Bonus:

7. $11115 \times 11117 =$ _____

Most Challenging Problem:

2015 University of Maryland High School Mathematics Competition Problem 20

Let p be the greatest prime factor of 9,991. The sum of the digits of p is equal to

 (A) 4   (B) 10   (C) 13   (D) 16   (E) 28

Hint to get started: $9{,}991 = 10{,}000 - 9 = 100^2 - 3^2$

Part II

Comprehensive Competitive Math Problem Set

Problem 1

Jay and Ray are each told to calculate $9 - (4 + 3)$. Jay gets the correct answer. Ray ignores the parentheses and calculates $9 - 4 + 3$. If Jay's answer is J and Ray's answer is R, what is $J - R$?

(A) -10 **(B)** -6 **(C)** 0 **(D)** 6 **(E)** 10

Problem 2

Jeffrey owes Jenifer 45 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?

(A) 6   (B) 7   (C) 8   (D) 9   (E) 10

Problem 3

Joshua had a week to read a book for a school assignment. He read an average of 43 pages per day for the first three days and an average of 57 pages per day for the next three days. He then finished the book by reading 40 pages on the last day. How many pages were in the book?

(A) 300   (B) 310   (C) 320   (D) 330   (E) 340

Problem 4

The sum of two prime numbers is 99. What is the product of these two prime numbers?

(A) 99   (B) 121   (C) 194   (D)205   (E) 231

Problem 5

Eva's car can go 40 miles on a gallon of gas, and gas currently costs $6 per gallon. How many miles can Eva drive on $48 worth of gas?

(A) 64   **(B)** 128   **(C)** 160   **(D)** 320   **(E)** 640

Problem 6

Five rectangles each with a common base width of 3 have lengths of 1, 4, 9, 16, and 25. What is the sum of the areas of the six rectangles?

(A) 55   (B) 58   (C) 84   (D)123   (E) 165

Problem 7

There are three more boys than girls in Mr. Bush's class of 21 students. What is the ratio of number of boys to the number of girls in his class?

**(A)** $3:4$ **(B)** $4:3$ **(C)** $3:2$ **(D)** $7:4$ **(E)** $2:1$

Problem 8

Eighteen members of the Middle School Science Club each paid the same amount for a guest speaker to talk about problem solving at their science club meeting. They paid their guest speaker $\underline{1X8}$. What is the missing digit X of this 3-digit number?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

Problem 9

In $\triangle ABC$, D is a point on side AB such that $AD = DC$ and $\angle CAD$ measures $70°$. What is the degree measure of $\angle BDC$?
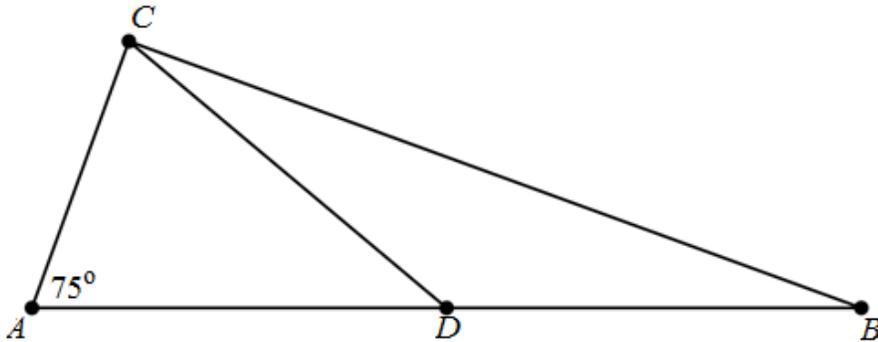
(A) 100   (B) 120   (C) 135   (D) 140   (E) 150

Problem 10

The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 11 years old the year that she took the ninth AMC 8. In what year was Samantha born?

(A) 1979   (B) 1980   (C) 1981   (D) 1982   (E) 1983

Problem 11

Daniel wants to bike from his house to David's house, which is located two blocks east and three blocks north of Daniel's house. After biking each block, Daniel can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach David's house by biking a total of five blocks?

(A) 4   (B) 5   (C) 6   (D) 8   (E) 10

Problem 12

Ramya prepared 3 different letters to be sent to 3 different addresses. For each letter she prepared an envelope with its correct address. If the 3 letters are to be put in 3 envelopes at random, what is the probability?

(A) $\dfrac{1}{9}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{1}{2}$
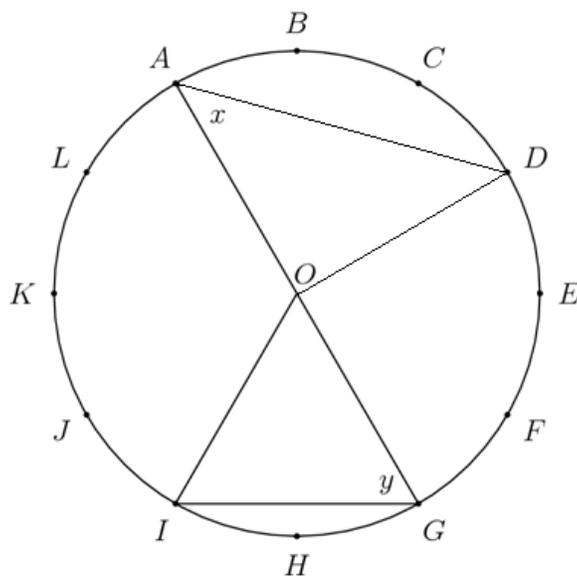
Problem 13

If a and b are integers and $a^2 + b^2$ is odd, which of the following is possible?

(A) a and b are odd   (B) a and b are even   (C) $a + b$ is odd

(D) $a + b$ is even   (E) none of these are possible

Problem 14

The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the difference of the angles x and y?

(A) 10   (B) 15   (C) 30   (D) 60   (E) 90

Problem 15

Right triangle ABC and rectangle BCDE have the same area. They are joined to form a trapezoid, as shown. What is AB?
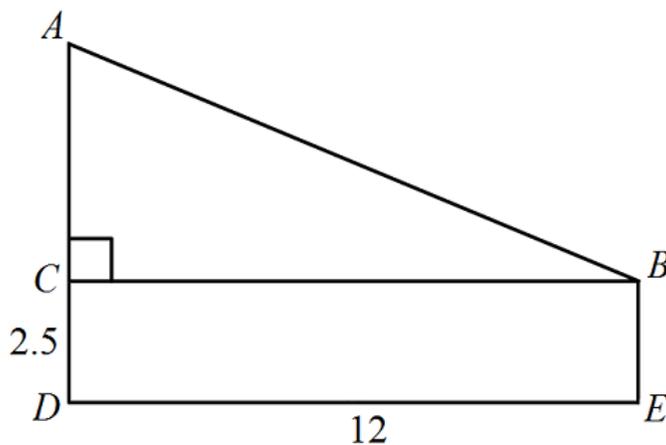
(A) 12   **(B)** 13   **(C)** 14   **(D)** 15   **(E)** 16

Bonus -- Challenging Problems:

Problem 16

The "Middle School Seven" baseball conference has 7 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 5 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Seven" teams?

(A) 49   (B) 56   (C) 77   (D) 119   (E) 133

Problem 17

Aaron walks 2 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first 1 mile at a speed of only 2 miles per hour. At how many miles per hour must Aaron run the last 1 mile in order to arrive just as school begins today?

(A) 6   (B) 8   (C) 10   (D) 12   (E) 14

Problem 18

If we toss four fair coins simultaneously, which of the following outcomes is most likely?

(A) getting 4 Heads   (B) getting 4 Tails   (C) getting exactly 2 Heads

(D) getting exactly either 3 Heads or 3 Tails   (E) all of these outcomes are equally likely

Problem 19

A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored white and 7 are colored black. If the 3-inch cube is constructed to have the smallest possible black surface area showing, what fraction of the surface area is black?

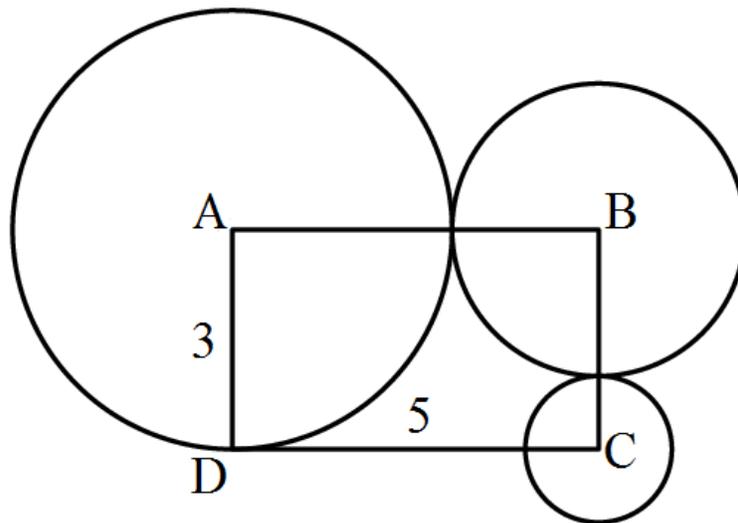(A) $\frac{1}{9}$ (B) $\frac{7}{54}$ (C) $\frac{1}{6}$ (D) $\frac{2}{9}$ (E) $\frac{7}{27}$

Problem 20

Rectangle ABCD has sides AD = 3 and DC = 5. A circle of radius 3 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 1 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

(A) 3.5   (B) 4.0   (C) 4.5   (D) 5.0   (E) 5.5

Bonus -- Most Challenging Problems:

Problem 21

The 8-digit numbers 851X43Y2 and 673X9Y2Z are each multiples of 3. Which of the following could be the value of Z?

(A) 2   (B) 3   (C) 4   (D) 6   (E) 7

Problem 22

A 2-digit number is such that the sum of the digits plus three times the product of the digits is equal to the number. What is the units digit of the number?

(A) 1   (B) 3   (C) 5   (D) 7   (E) 9

Problem 23

Three members of the Takoma Park Middle School boys' basketball team had the following conversation.

Alex: I just realized that our uniform numbers are all 2-digit primes.

Bill: And the sum of your two uniform numbers is the date of my birthday earlier this month.

Cole: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Alex: And the sum of you two uniform numbers is today's date.

What number does Cole wear?

(**A**) 11   (**B**) 13   (**C**) 17   (**D**) 19   (**E**) 23
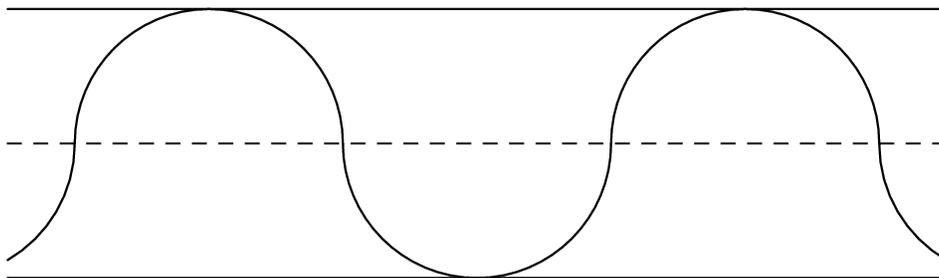
Problem 24

One day the Beverage Barn sold 351 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the minimum possible median number of cans of soda bought per customer on that day?

(**A**) 2.5   (**B**) 3.0   (**C**) 3.5   (**D**) 4.0   (**E**) 4.5

Problem 25

A straight two-mile stretch of highway, 60 feet wide, is closed. Alison rides his bike on a path composed of semicircles as shown. If she rides at 4 miles per hour, how many hours will it take to cover the two-mile stretch?

Note: 1 mile= 5280 feet

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{10}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{4}$

Part III:

Specific Problem Set

Topic: Modular Arithmetic

Problem 1: Pretend that it's **3:00** now. Answer the following questions, but don't worry about AM/PM.

(a) In 17 hours, what time will the clock show?

(b) In 33 hours, what time will the clock show?

(c) What time did the clock show 15 hours ago?

(d) What time will the clock read 17 hours after the time it shows 19 hours before 4:00?

Problem 2: Today is Thursday. Answer the following questions.

(a) What day of the week will it be 5 days from now?

(b) What day of the week will it be 17 days from now?

(c) What day of the week was it 10 days ago?

Problem 3: Solve the following:

(a) What is 84 (mod 9)?

(b) What is 52 (mod 5)?

(c) What is -4 (mod 10)?

Problem 4: Create the following tables:

(a) Addition table for modulo 7

(b) Multiplication table for modulo 7

NOTE: A *leap year* occurs every four years. A leap year occurs on any year that is divisible by 4 (ex: 4, 8, 12 ... 1996, 2000, 2004, 2008, 2012, 2016).

Problem 5: I celebrated my 21st birthday on Wednesday, July 27th, 2011. On what day of the week was I born? (Don't forget about leap years!)

Problem 6: One year on Venus lasts 225 Earth days. Eva is 13 years and 83 days old. How many days until her next Venusian birthday? How old will she be turning (in Venusian years)? Omit leap years for simplicity.

Problem 7: It is 8:00 AM in our 24 hour world. What time is it in a 3 hour world?

Problem 8: Using a standard 52 card deck, Leo deals all the cards out to Ray, Tom, and himself. Were the cards dealt evenly?

Problem 9: Dale is facing West, he rotates 1260° clockwise. What direction is he now facing? (Note: A circle has 360 degrees)

Bonus -- Challenging Problems:

Problem 10: 1 year on Jupiter is equal to approximately 12 Earth years. On what day of the week did you celebrate your 1st Jovian (or Jupiterian) birthday? (If you haven't turned 1 on Jupiter yet, calculate on which day of the week your 1st birthday will fall)

Problem 11: Tim counted the loonies in her pocket. When she put them in groups of 4, she had 2 loonies left over. When she put them in groups of 5, she had one loonie left over. If Philippa has more than 10 loonies, what is the smallest possible number of loonies she could have?